Exam :: Conférences de méthodes en Économie du Développement

Exam instructions:

- Closed book exam: supporting materials are not allowed.
- Total time: 2 hours.
- When answering the questions below, please try to focus on the topic being asked.
- You may answer in English or French.
- Question 3 is optional.
- Please, choose between Question 4 and Question 5 and answer it.

Question 1 (10 points):

Suppose the basic OLG model we have seen in class. In particular, assume a large number of identical firms producing a single, homogenous good and using the same technology. These firms operate under perfect competition and thus make zero profits. Production requires physical capital and labour, which are combined using a Cobb-Douglas production function such that, at every period of time:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \alpha \in (0,1),$$

where Y_t denotes total production, K_t represents physical capital and L_t labour.

This economy is populated by a large number of individuals who live for two periods. During the first period of life, agents are young and supply one unit of labour inelastically and receive the ongoing wage rate w_t. Young agents optimally allocate the wage between consumption when young c_t and saving s_t. During the second period of life agents are old and *do not work*. They use their savings (plus the interest rate R_{t+1}) to finance their consumption when old, denoted by d_{t+1}. Every period of time, a new generation is born. In particular, we assume that each old agent has $n > -1$ children. Individuals have perfect foresight. Finally, the utility of an agent can be represented by the following function:

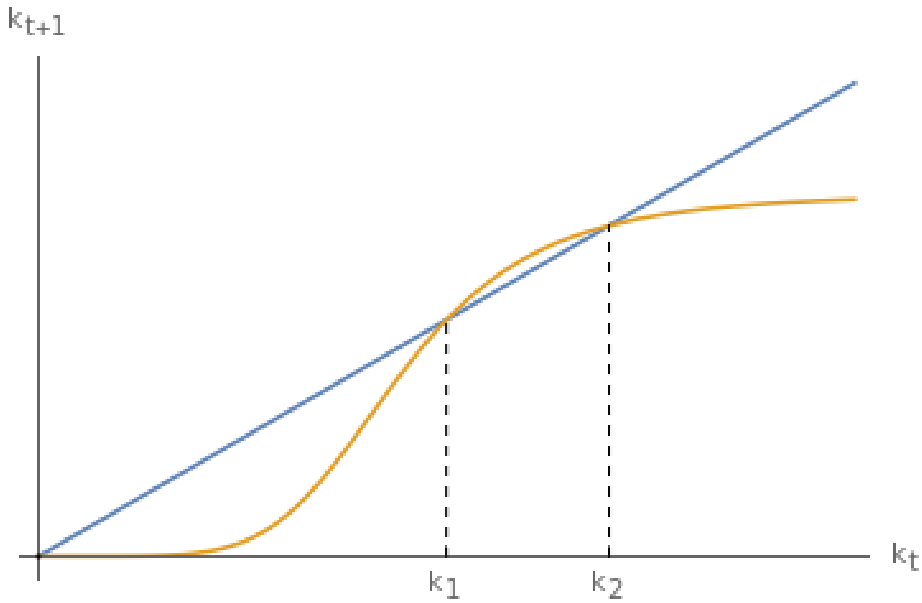
$$U(c_t, d_{t+1} = \log(c_t) + \beta \log(d_{t+1}), \beta \in (0,1).$$

- 1.1 Write the production function in intensive terms, this is, in terms of capital per worker, denoted by $k_t \equiv \frac{K_t}{L_t}$ and obtain the wage and interest rate as a function of k_t.
- 1.2 Write the utility maximisation problem face by an individual born at period t.
- 1.3 Derive the Euler equation and obtain the savings function $s_t = s\left(w(k_t), R_{t+1}\right)$.
- 1.4 Using the saving function, compute the intertemporal equilibrium, this is, the equation that relates k_{t+1} to k_t.
- 1.5 Discuss whether the intertemporal equilibrium is unique or not.

- 1.6 Draw a diagram illustrating the relationship between k_t and k_{t+1} and indicate *all* the steady states.
- 1.7 Discuss the stability of each steady state.

Question 2 (5 points):

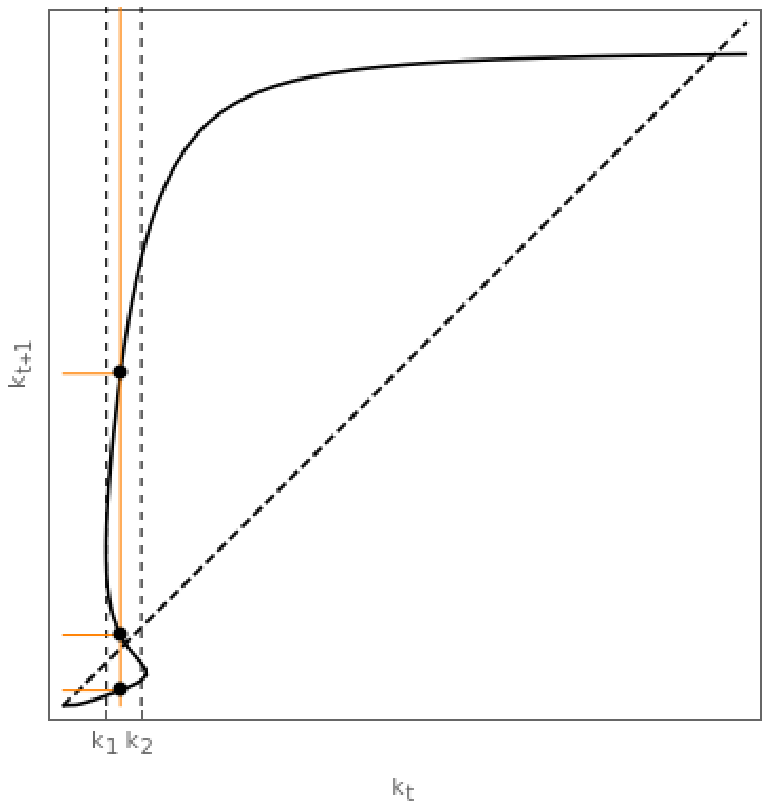
Suppose that, given the preferences of the agents and the production, the evolution of capital in an OLG model can be represented by the following diagram:



Identify the steady states that exist in this economy, discuss their stability and relate both to the concept of poverty trap.

Question 3 (optional, 5 points):

In class, we discussed that it is possible for the OLG model to feature a multiplicity of intertemporal equilibria. The following diagram illustrates one such possibility: for levels of capital k_t, multiple values k_{t+1} exist. Explain its implications for the OLG model.



Question 4 (de la Croix and Dottori, 2008) (5 points):

De la Croix and Dottori (2008) illustrate how competition for resources could have had disastrous effects in Easter Island. Describe their model's general set-up and explain why a population race occurs, relating it to the non-cooperative bargaining process that is implemented.

Question 5 (Galor and Moav, 2006) (5 points):

Galor and Moav (2006) present a theoretical model that is capable of explaining the rise of publicly financed education through taxes. In particular, as time advances, capitalists find it optimal to tax themselves and use the proceedings to finance education for *all* individuals. Describe the general set-up of the model and explain in detail why the above paradox is, in fact, rational.