Quick tips before trying to solve the OLG model.

1. Determine whether the zero steady state is possible:
   If $f(0) = 0$ (or, equivalently, $w(0) = 0$), then the zero (autarky) steady state exists. It may be stable or unstable, but it exists.

   - Cobb-Douglas production function:
     $f(k) = k^\alpha \implies f(0) = 0$
   - CES production function:
     Important remark: verify the exact functional form of the production function, in particular how the exponents are written.
     $f(k) = (\alpha k^\rho + (1-\alpha))^{\frac{1}{\rho}}$
     - Complementary inputs ($\rho < 0$)
       $\lim_{k \to 0} f(k) = 0$, assuming $\rho < 0, \alpha \in (0,1)$.
     - Substitue inputs ($\rho > 0$)
       $\lim_{k \to 0} f(k) = (1-\alpha)^{\frac{1}{\rho}}$, assuming $\rho > 0, \alpha \in (0,1)$.

2. Determine whether the interest rate appears in the savings function:

   - If log-utility, the interest rate does not appear.
     $u(c) = \log(c) \implies u'(c) = \frac{1}{c}$
     The savings function is obtained in this case by solving:
     $u'(w-s) = \beta R u'(Rs)$ so $s = \frac{\beta}{1+\beta} w$
   - Under a CIES, R appears in the savings function:
     $u'(c) = x^{\frac{-1}{\sigma}}$
     The savings function is obtained in this case by solving:
     $(w-s)^{\frac{-1}{\sigma}} = \beta R (Rs)^{\frac{-1}{\sigma}}$
     $s = \frac{w}{1+\beta^{-\sigma} R^{1-\sigma}}$

Examples

Example 1

Production function: $f(k) = k^{0.5}$
Utility function: $u(c) = 2\sqrt{c}$
Savings function: $s(w(k_t), f'(k_{t+1})) = \frac{16.Rw}{25.+16.R}$
Substituting: $s(w(k_t), f'(k_{t+1})) = \frac{4.}{25.+\frac{8.}{k^{0.5}}}$
Steady States: Solve $k_{t+1} = \frac{1}{1+n} s(w(k_t), f'(k_{t+1}))$
With the parameters: $\{k \to 0.\}, \{k \to 0.0733398\}$
Stability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state.
With the parameters:
Steady state 1, Capital level: 0. Derivative=∞ Unstable.
Steady state 2, Capital level: 0.0733398 Derivative=0.406773 Stable.

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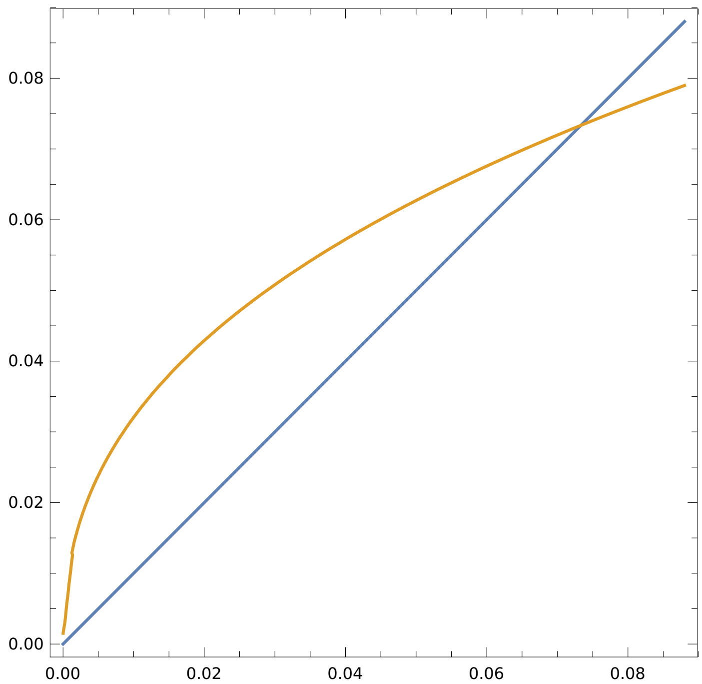

Example 2

Production function: $f(k) = \frac{1}{\sqrt{0.5+\frac{0.5}{k^2}}}$

Utility function: $u(c) = 2\sqrt{c}$

Savings function: $s(w(k_t), f'(k_{t+1})) = \frac{16.Rw}{25.+16.R}$

Substituting: $s(w(k_t), f'(k_{t+1})) = \frac{8.\left(\frac{1}{\sqrt{0.5+\frac{0.5}{k^2}}} - \frac{0.5}{\left(0.5+\frac{0.5}{k^2}\right)^{3/2}k^2}\right)}{\left(25.+\frac{8.}{\left(0.5+\frac{0.5}{k^2}\right)^{3/2}k^3}\right)\left(0.5+\frac{0.5}{k^2}\right)^{3/2}k^3}$

Steady States: Solve $k_{t+1} = \frac{1}{(1+n)} s(w(k_t), f'(k_{t+1}))$

With the parameters: $\{k \to 0\}\}$

Stability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state.

With the parameters:

Steady state 1, Capital level: 0, Derivative=0. Stable.

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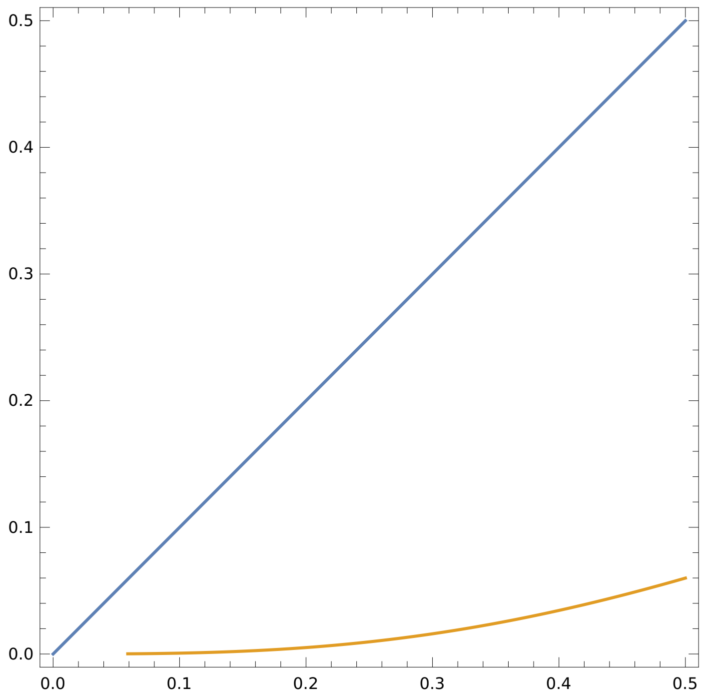

Examples 3

Production function: $f(k) = \frac{1}{\sqrt{0.5+\frac{0.5}{k^2}}}$
Utility function: $u(c) = \log(c)$
Savings function: $s(w(k_t), f'(k_{t+1})) = 0.444444w$
Substituting: $s(w(k_t), f'(k_{t+1})) = 0.444444\left(\frac{1}{\sqrt{0.5+\frac{0.5}{k^2}}} - \frac{0.5}{\left(0.5+\frac{0.5}{k^2}\right)^{3/2}k^2}\right)$
Steady States: Solve $k_{t+1} = \frac{1}{(1+n)}s(w(k_t), f'(k_{t+1}))$
With the parameters: $\{\{k \to 0\}\}$
Stability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state.
With the parameters:
  Steady state 1, Capital level: 0 Derivative=0. Stable.

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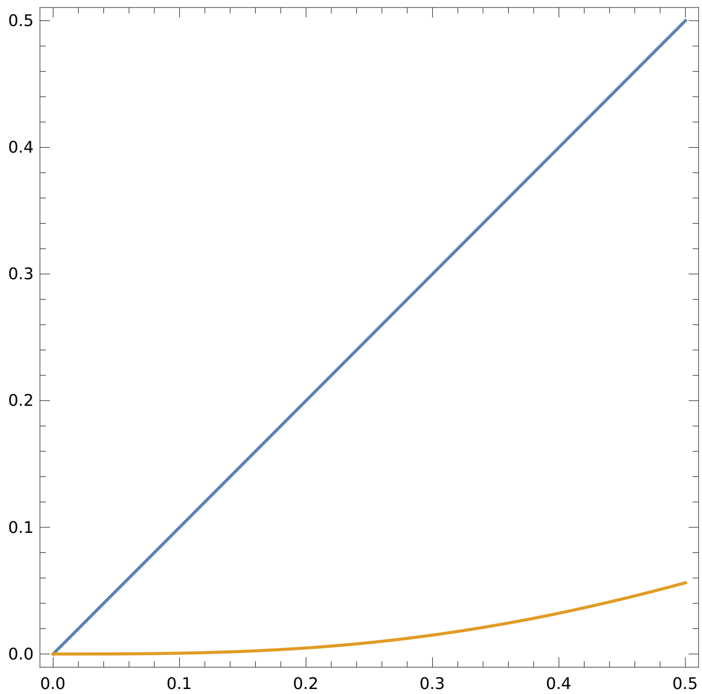

Example 4

Production function: $f(k) = \left(0.5\ + 0.5k^{0.5}\right)^{2.}$
Utility function: $u(c)log(c)$
Savings function: $s(w(k_t), f'(k_{t+1})) = 0.444444w$
Substituting: $s(w(k_t), f^prime(k_{t+1})) = 0.444444\left(\left(0.5\ + 0.5k^{0.5}\right)^{2.} - 0.5\left(0.5\ + 0.5k^{0.5}\right)^{1.}k^{0.5}\right)$
Steady States: Solve $k_{t+1} = \frac{1}{(1+n)}s(w(k_t), f'(k_{t+1}))$
With the parameters: $\{k \to 0.154832\}\}$
Stability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state.
With the parameters:

Steady state 1, Capital level: 0.154832 Derivative=0.141188 Stable.

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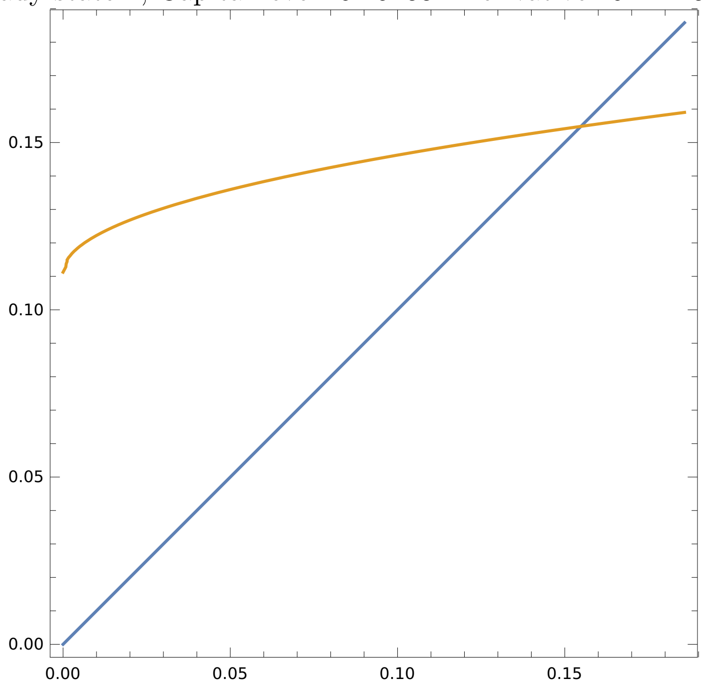

Example 5

Production function: $f(k) = \left(0.5\ + 0.5k^{0.5}\right)^{2.}$
Utility function: $u(c) = 2\sqrt{c}$
Savings function: $s(w(k_t), f'(k_{t+1})) = \frac{16.Rw}{25.+16.R}$
Substituting: $s(w(k_t), f'(k_{t+1})) = \frac{\left(8.\left(0.5+0.5k^{0.5}\right)^{1.}\left(\left(0.5+0.5k^{0.5}\right)^{2.}-0.5\left(0.5+0.5k^{0.5}\right)^{1.}k^{0.5}\right)\right)}{\left(\left(25.+\frac{8.\left(0.5+0.5k^{0.5}\right)^{1.}}{k^{0.5}}\right)k^{0.5}\right)}$
Steady States: Solve $k_{t+1} = \frac{1}{(1+n)} s(w(k_t), f'(k_{t+1}))$
With the parameters: $\{k \to 0.128222\}$
Stability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state.
With the parameters:
  Steady state 1, Capital level: 0.128222 Derivative=0.107258 Stable.

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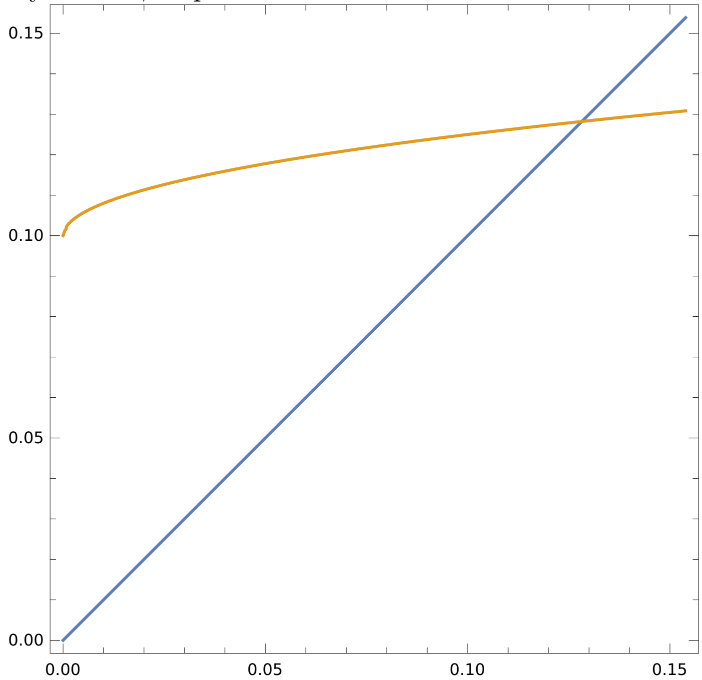

Example 6

Production function: $f(k) = k^{0.5}$
Utility function: $u(c) = \log(c)$
Savings function: $s(w(k_t), f'(k_{t+1})) = 0.444444w$
Substituting: $s(w(k_t), if'(k_{t+1})) = 0.222222k^{0.5}$
Steady States: Solve $k_{t+1} = \frac{1}{(1+n)} s(w(k_t), f'(k_{t+1}))$
With the parameters: $\{k \to 0.\}, \{k \to 0.0493827\}$
Stability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state.
With the parameters:
  Steady state 1, Capital level: 0. Derivative=∞ Unstable.
  Steady state 2, Capital level: 0.0493827 Derivative=0.5 Stable.

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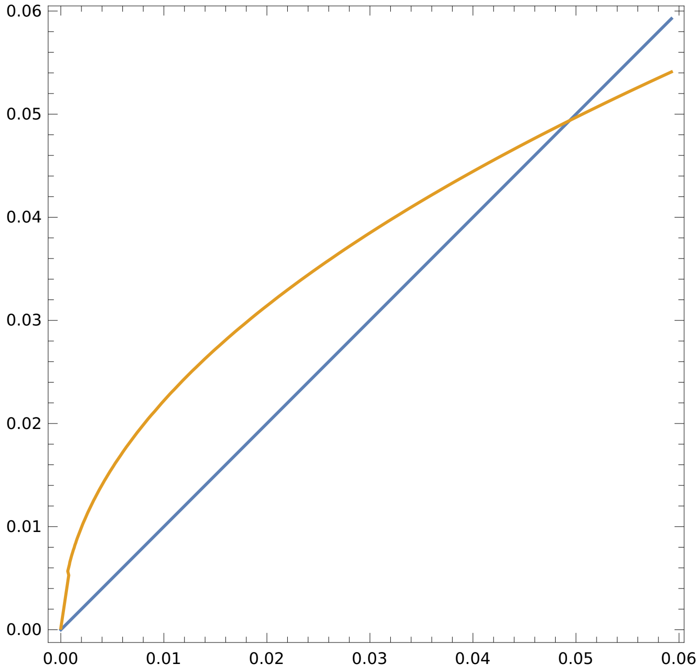

Example 7

Production function: $f(k) = \frac{1}{0.9+\frac{0.1}{k}}$

Utility function: $u(c) = 2\sqrt{c}$

Savings function: $s(w(k_t), f'(k_{t+1})) = \frac{16.Rw}{25.+16.R}$

Substituting: $s(w(k_t), f'(k_{t+1}) = \frac{1.6\left(\frac{1}{0.9+\frac{0.1}{k}} - \frac{0.1}{\left(0.9+\frac{0.1}{k}\right)^2 k}\right)}{\left(25.+\frac{1.6}{\left(0.9+\frac{0.1}{k}\right)^2 k^2}\right)\left(0.9+\frac{0.1}{k}\right)^2 k^2}$

Steady States: Solve $k_{t+1} = \frac{1}{(1+n)} s(w(k_t), f'(k_{t+1}))$

With the parameters: $\{k \to 0.018251\}, \{k \to 0.206304\}$

Stability: Compute the derivative of k_{t+1} with respect to k_t and evaluate at each steady state.

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